4.2 Solve Multi-Step Equations

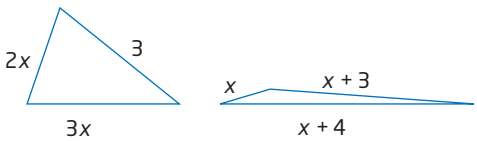
Look at the shapes in the bridge. The beams that form the triangular structures are called trusses. These are very useful in engineering and architecture, because they add strength and stability to structures such as bridges and buildings.

Where else are you likely to find triangles in common settings? How can equations be used to describe geometric relationships and to solve problems?

Investigate

How can you use equations to model and solve problems?

These two triangles have the same perimeter.



You can use the equation $5x + 3 = 3x + 7$ to model the situation and find the side lengths of the two triangles.

Making Connections

Perimeter is the distance around the outside of a shape. Think how each side of this equation was found.

1. Solve the equation by isolating the variable on the left side of the equation. Explain your steps in words and algebraic symbols.
2. Solve the equation by isolating the variable on the right side of the equation. Explain your steps in words and algebraic symbols.
3. **Reflect**
   - **a)** Compare your results from steps 1 and 2.
   - **b)** Explain how you can collect like terms to solve equations with the variable on both sides.
4. Describe how you would use the solution to the equation to find the side lengths of each triangle.

Example 1 Solve Equations by Collecting Like Terms

Solve.

a) $3x + 2 = 2x - 4$ **b)** $7 - 2k = 8 - 5k$ **c)** $5 - 3m = -2 - 2m$

Solution

To solve an equation involving several terms, collect variable terms on one side of the equation and **constant terms** on the other.

a)

$$\begin{aligned} 3x + 2 &= 2x - 4 \\ 3x + 2 - 2x &= 2x - 4 - 2x \\ x + 2 &= -4 \\ x + 2 - 2 &= -4 - 2 \\ x &= -6 \end{aligned}$$

The variable terms are $3x$ and $2x$. I'll collect these on the left side by subtracting $2x$ from each side.

The constant terms are 2 and −4. I'll collect these on the right side by subtracting 2 from each side.

b) You can collect variable terms and constant terms in one step.

Method 1: Pencil and Paper

$$\begin{aligned} 7 - 2k &= 8 - 5k \\ 7 - 2k + 5k - 7 &= 8 - 5k + 5k - 7 \\ -2k + 5k &= 8 - 7 \\ 3k &= 1 \\ \frac{3k}{3} &= \frac{1}{3} \\ k &= \frac{1}{3} \end{aligned}$$

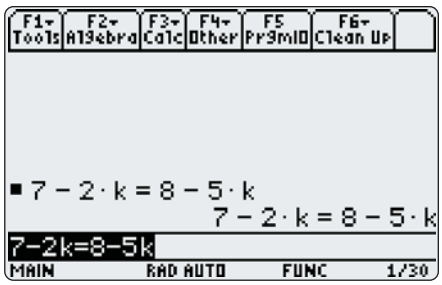
Divide both sides by 3.

I'll collect variable terms on the left side by adding $5k$ to both sides.

I'll collect constant terms on the right side by subtracting 7 from both sides.

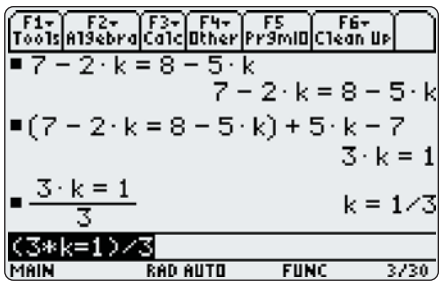
Method 2: Computer Algebra System (CAS)

In the Home screen, type the equation $7 - 2k = 8 - 5k$.
Press ENTER.



To collect variable terms on the left side and constant terms on the right, add $5k$ and subtract 7:

- Put brackets around the equation.
- Type $+ 5k - 7$.
- Press ENTER.



Making Connections

You learned how to collect like terms in Chapter 3.

constant term

- a term that does not include a variable
- in $2x + 5$, the constant term is 5

Literacy Connections

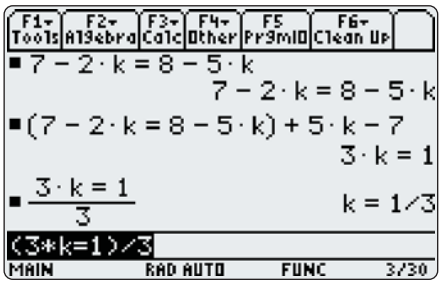
A constant term is called constant because its value does not change.

Technology Tip

Use the **ALPHA** key to enter variables other than **X**, **Y**, **Z**, or **T**. For example, to enter the variable k, press ALPHA EE for [K].

COPY and PASTE the resulting equation in the command line.
Put brackets around the equation.
Divide by 3 to find the solution.

$k = \frac{1}{3}$



c) When you solve an equation, it does not matter which side you isolate the variable on.

Method 1: Isolate *m* on the Left Side

$$\begin{aligned} 5 - 3m &= -2 - 2m \\ 5 - 3m + 2m - 5 &= -2 - 2m + 2m - 5 && \text{Add } 2m \text{ to both sides and subtract 5 from both sides.} \\ -3m + 2m &= -2 - 5 \\ -m &= -7 \\ \frac{-m}{-1} &= \frac{-7}{-1} && \text{Divide both sides by } -1. \\ m &= 7 \end{aligned}$$

Method 2: Isolate *m* on the Right Side

$$\begin{aligned} 5 - 3m &= -2 - 2m \\ 5 - 3m + 3m + 2 &= -2 - 2m + 3m + 2 && \text{Add } 3m \text{ and add 2 to both sides.} \\ 5 + 2 &= -2m + 3m \\ 7 &= m \end{aligned}$$

Both methods give the solution, $m = 7$. In this case, isolating the variable on the right side saves a step.

Example 2 Solve Equations With Brackets

a) Solve. $5(y - 3) - (y - 2) = 19$

b) Solve and check. $2(x - 3) = -3(x + 5) - 6$

Solution

a) Expand to remove the brackets. Then, collect like terms.

$$\begin{aligned} 5(y - 3) - (y - 2) &= 19 \\ 5(y - 3) - 1(y - 2) &= 19 && \text{Apply the distributive property to remove brackets.} \\ 5y - 15 - y + 2 &= 19 \\ 4y - 13 &= 19 \\ 4y - 13 + 13 &= 19 + 13 && \text{Add 13 to both sides.} \\ 4y &= 32 \\ \frac{4y}{4} &= \frac{32}{4} && \text{Divide both sides by 4.} \\ y &= 8 \end{aligned}$$

b)

$$2(x-3) = -3(x+5) - 6$$ Apply the distributive property to remove brackets.

$$2x - 6 = -3x - 15 - 6$$

$$2x - 6 = -3x - 21$$

$$2x - 6 + 6 = -3x - 21 + 6$$ Add 6 to both sides.

$$2x = -3x - 15$$

$$2x + 3x = -3x - 15 + 3x$$ Add 3x to both sides.

$$5x = -15$$

$$\frac{5x}{5} = \frac{-15}{5}$$ Divide both sides by 5.

$$x = -3$$

Check by substituting $x = -3$ into each side of the equation. Both sides must have the same value for this solution to be correct.

L.S. $= 2(x-3)$
$= 2[(-3) - 3]$
$= 2(-6)$
$= -12$

R.S. $= -3(x+5) - 6$
$= -3[(-3) + 5] - 6$
$= -3(2) - 6$
$= -6 - 6$
$= -12$

L.S. = R.S.

Since L.S. = R.S., $x = -3$ is the solution, or root, of the equation.

Literacy Connections

Follow these guidelines when checking a solution (root):

- Use nested brackets when substituting, where necessary. For example: $2(x-3)$ when $x = -3$ $= 2[(-3) - 3]$
- Follow the correct order of operations. Use BEDMAS to help you remember the order.
- Remember the rules for integer operations.

Example 3 Use an Equation to Model a Geometric Relationship

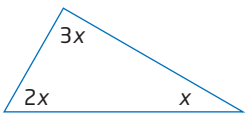
A triangle has angle measures that are related as follows:

- the largest angle is triple the smallest angle
- the middle angle is double the smallest angle

Find the measures of the angles.

Solution

Let x represent the smallest angle. The other angles are double and triple this value: $2x$ and $3x$.



The sum of the three interior angles of a triangle is 180°.

$$x + 2x + 3x = 180°$$

$$6x = 180°$$

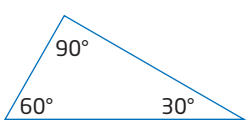
$$\frac{6x}{6} = \frac{180°}{6}$$ Divide both sides by 6.

$$x = 30°$$

Have I answered the question? I have only found x, which is the smallest angle. I still need to find the other two angles.

$2x$
$= 2(30°)$
$= 60°$

$3x$
$= 3(30°)$
$= 90°$



The three angles are 30°, 60°, and 90°.

Key Concepts

- To solve an equation involving multiple terms, collect variable terms on one side of the equation and constant terms on the other.
- To solve an equation involving brackets, you may need to expand the brackets first.
- Check a solution by substituting the root into the left side and right side of the original equation. Both sides must have the same value.

Communicate Your Understanding

C1 Describe the first step you would take to solve each equation.

a) $2x + 7 = 4x - 9$

b) $13 = -8 + 3m$

c) $3(k - 2) = 2(k + 8)$

C2 $p = -2$ is the correct solution to which equation? Explain how you can tell without solving the equations.

A $6p - 5 = 8p - 9$

B $4(p - 1) = -7(p + 4)$

C $3(p - 1) = -6 - (p + 5)$

Practise

For help with questions 1 to 3, see Example 1.

1. Solve using pencil and paper.

a) $3 + 4m + 5m = 21$

b) $16y - 8 - 9y = 27$

c) $46 = 2 - 8w - 3w$

d) $3d + 4 - 9d + 12 = 0$

2. Solve using pencil and paper.

a) $5x + 9 = 3x + 7$

b) $-2u - 8 = 5u - 1$

c) $4y - 13 = -6y + 7$

d) $7 - 5m = -2 - 2m$

3. Solve using a CAS. Use at least two steps.

a) $0 = 14 - x + 6x - 9$

b) $11 - n + 3 = 3n + 3n$

c) $4t - 5 = 2t + 5$

d) $6k - 3 - 2k = k - 3$

For help with questions 4 and 5, see Example 2.

4. Find the root of each equation using pencil and paper.

a) $2(x - 2) = 4x - 2$

b) $4c + 3 = 3(c - 4)$

c) $6p + 4(8 - p) = 22$

d) $k = 2(11 - k) + 14$

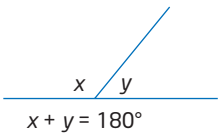
5. Find the root of each equation using pencil and paper. Check each solution.

a) $2(x - 3) + 3(x - 2) = 18$ **b)** $4(y - 1) - (y - 5) = 10$

c) $2(c + 2) = 5(c + 1) - 7$ **d)** $3(t - 4) = -2(t + 3) + 14$

For help with questions 6 and 7, see Example 3.

6. Two or more angles are supplementary if their sum is 180°.

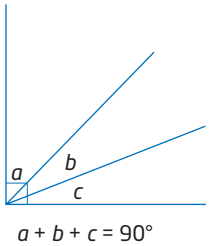


a) An angle is twice the size of its supplement. Set up and solve an equation to find the measures of the two angles.

b) An angle is five times the value of its supplement. Find the measures of the two angles.

7. Two or more angles are complementary if their sum is 90°.

Three angles are complementary. One angle is double the smallest angle. The largest angle is triple the smallest angle. Find the measures of the three angles.



Making Connections

You will study geometric relationships in more depth in Chapter 7: Geometric Relations.

Connect and Apply

8. The following shows that $x = -3$ is the correct solution to the equation $2(x + 4) + 5 = 6 - (x + 2)$. Copy this check and explain each step. The first step has been done for you.

Step	*Explanation*
L.S. $= 2[(-3) + 4] + 5$ $= 2(1) + 5$ $= 2 + 5$ $= 7$	Substitute the root into the left side.
R.S. $= 6 - [(-3) + 2]$ $= 6 - (-1)$ $= 6 + 1$ $= 7$	
L.S. = R.S.	
Therefore, $x = -3$ is correct.	

9. Solve each equation. Express fraction answers in lowest terms. Check each solution.

a) $3x - 8 = 7x + 10$ **b)** $3 + 10i = 4i - 18$

c) $-4(u + 6) = 2(3u - 4)$ **d)** $4(k - 3) = 2 - (2k - 6)$

e) $3(p + 7) - (4p - 1) = -5(2p - 3) + 1$

f) $8 - (3w - 2) = -5(w - 3) - (4w - 3)$

10. Find the root, to one decimal place. Check each answer.

a) $3.2x - 7.4 = 2.1x + 1.5$

b) $3(2.5d - 1.1) = 2(5.2 - 3.3d)$

11. How does a Computer Algebra System handle complicated equations? Refer to Example 2, part a):

$5(y - 3) - (y - 2) = 19$

a) Enter this equation into the Home screen and press ENTER.

b) Write down the screen output.

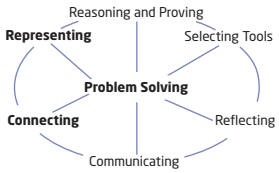
c) Explain what the CAS did to the equation.

d) Use the CAS to finish solving the equation.

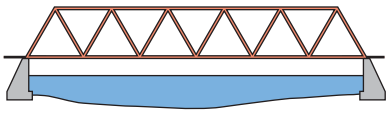
12. Use a CAS to find the root of each equation.

a) $(5q - 2) + (3 - 4q) = 4$

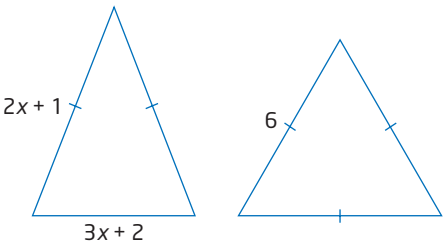
b) $17 = (6u + 7) - (3u - 10)$



13. One type of truss design commonly used to build bridges is known as the Warren truss pattern. This features a series of equilateral or isosceles triangles.



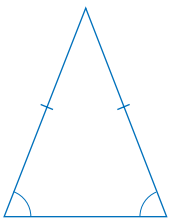
An isosceles triangle and an equilateral triangle have the same perimeter. Find the side lengths of each triangle.



Literacy Connections

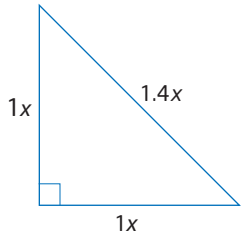
An isosceles triangle has two equal sides and two equal angles.

14. A family of isosceles triangles has the property that the two equal angles are each half the value of the third angle. Find the measures of the angles.



Extend

15. A family of right isosceles triangles has side lengths in the approximate ratio 1:1:1.4. A triangle belonging to this family has a perimeter of 50 cm.



a) Find the length of each side, to the nearest tenth of a centimetre.

b) Explain how you solved this.

16. A family of right triangles has side lengths in the approximate ratio 1:1.7:2. One triangle belonging to this family has a perimeter of 100 cm. Find its area, to the nearest square centimetre.

17. Solve each equation. Express fraction answers in lowest terms.

a) $\frac{1}{2}(x + 6) = 4(x - 2)$

b) $\frac{1}{3}k + \frac{1}{2} = \frac{1}{4}k$

18. **Math Contest** Solve. Check your solutions.

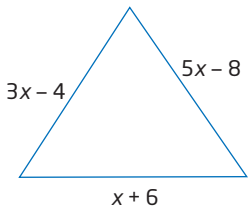
a) $x(x - 12) = 30 + x(x + 3)$

b) $14 - x(x + 3) = 2x - x(x - 6) + 8$

19. **Math Contest** If $x = -4$ and $y = 3$ satisfy the equation $3x^2 + ky^2 = 24$, then which is the value of k?

A $\frac{1}{2}$ **B** $-\frac{1}{2}$ **C** 8 **D** $\frac{8}{3}$ **E** $-\frac{8}{3}$

20. **Math Contest** How many possible values of x make the triangle isosceles?



A 0

B 1

C 2

D 3

E more than 3

21. **Math Contest** Is there a value of x that makes this triangle equilateral? Explain your decision.